Exercises 2:
Evaluation of single exposured transit photos
by means of professional photos of Mercury's transit 2003

(with solutions)

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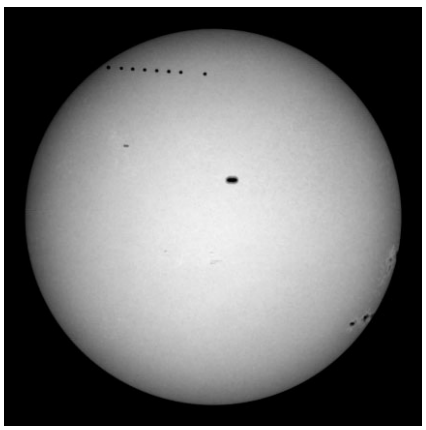
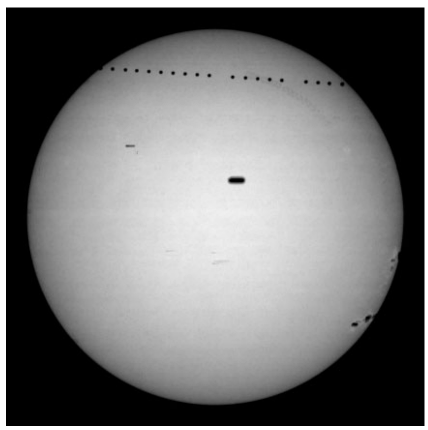
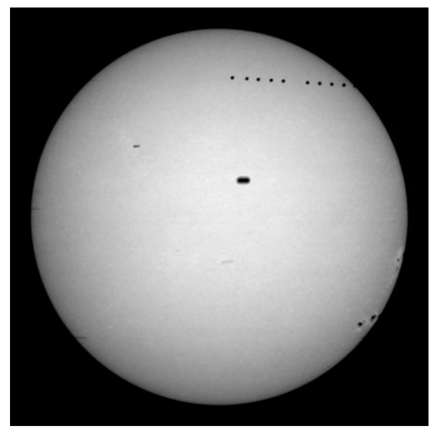




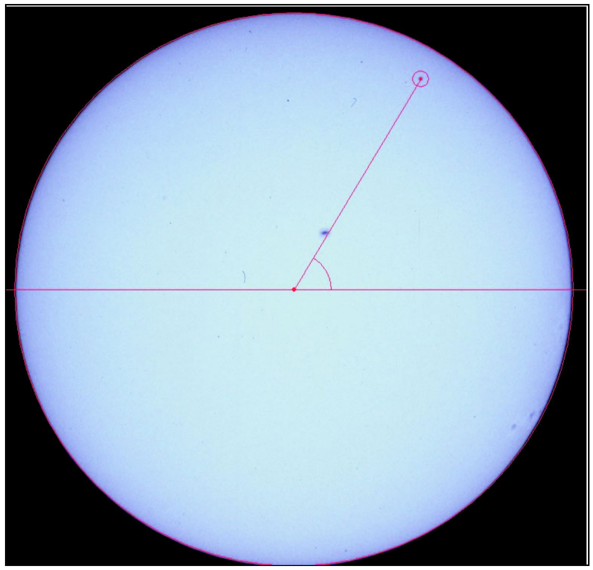
This exercise makes concrete the procedure of determining the distance to the Sun by evaluating transit photos. The procedure is described and explicated in the basic paper of this project[1].

The material consists of three series of photos of the transit of Mercury on May 7th, 2003[2], which have been published in the world wide web by the GONG network (http://gong.nso.edu/mercury_transit03). To each position of Mercury, the moments of exposure have been inserted into the original photos which have been taken by the following observatories:

- **El Teide, Canary Islands, Spain** ($\boldsymbol{\varphi = 28.3°,\ \lambda = -16°30'3''}$)
- **Udaipur, India** ($\boldsymbol{\varphi = 24°35.1',\ \lambda = 73°42.8'}$)
- **Learmonth, Australia** ($\boldsymbol{\varphi = -22.2°,\ \lambda = 114.1°}$)

Exercise 1 Determine the relative distances ρ' of all positions of Mercury from the center of the Sun's disc and the related position angles θ'.

[1] http://didaktik.physik.uni-essen.de/~backhaus/Venusproject/TransitEngl.pdf
[2] http://didaktik.physik.uni-essen.de/~backhaus/Venusproject/stuff/GONG.zip



Position measurement with `Bildauswertung.exe`

This work will become quite easy by using the evaluation program `Bildauswertung.exe`[3]. It makes it possible to fit circles to the solar disc and to the projection of Venus. From the so determined centers it calculates ρ' and θ'.

Exercise 2 From the only directly comparable photos, determine the parallactic shift β between Udaipur and Teide at 10.01 UT. First of all calculate the rectangular coordinates x'_U, y'_U, x'_T, y'_T. In order to be able to derive the parallactic shift you need to know the angular radius of the Sun ($\boldsymbol{\rho_S = 15.85'}$).

$$\left.\begin{array}{rl} x'_U = & 0.6094 \\ y'_U = & -70778 \\ x'_T = & 0.59385 \\ y'_T = & -0.70397 \end{array}\right\} \quad \Longrightarrow \quad \beta = 15.2''$$

Exercise 3 From β, derive the solar parallax π_S by using the relation

$$\pi_S = \left[\frac{R_E}{\Delta}\frac{1}{\sin w}\left(\frac{r_E}{r_M} - 1\right)\right]\beta.$$

(a) Derive the linear distance $\frac{\Delta}{R_E}$ between the both observatories from their geographical coordinates.

$$\frac{\Delta}{R_E} = 1.27$$

(b) Calculate the angle of projection w:

[3] `http://didaktik.physik.uni-essen.de/~backhaus/Venusproject/stuff/programs.zip`

i. Determine the equatorial coordinates which the observatories had when they took the pictures. For that you need the sideral time of Greenwich at 0.00 UT ($\boldsymbol{\Theta_{Gr_0} = 14h57m43s}$).

$$\alpha_U = 5h55m10s, \qquad \delta_E = 24.6^\circ$$
$$\alpha_T = 23h54m22s, \qquad \delta_W = 28.3^\circ$$

ii. Calculate the unit vector pointing to the Sun ($\boldsymbol{\alpha_S = 2h55m23s,\ \delta_S = 16^\circ 43'}$) and the unit vector between the both sites.

$$\vec{e}_S = (0.69, 0.66, 0.29)$$
$$\vec{e}_{UT} = (0.68, -0.73, 0.05)$$

iii. You can find w by forming the scalar product of these two unit vectors!

$$w = 90.3^\circ$$

How large is the projected distance $\frac{\Delta \sin w}{R_E}$ between the both sites?

$$\frac{\Delta \sin w}{R_E} = 1.27 \qquad (1)$$

(c) Now you can calculate the solar parallax π_S! For that you still need to know Mercury's relative distance from the Sun ($\boldsymbol{\frac{r_M}{r_E} = 0.446}$).

$$\pi_S = 14.9''$$

Exercise 4 Inspite of the very good pictures, the result of exercise 3 is not quite satisfying. To get a better measure it is necessary to minimize errors by a statistical method:

(a) Put the position which you have determined in exercise 1 into two Excel tabulars[4] and calculate the linear fits!

$$x_U = 0.087218 + 0.0042745 * t$$
$$y_U = 0.7442 - 0.000281313 * t$$
$$x_T = 0.07778 + 0.004262 * t$$
$$y_T = 0.7376 - 0.0002689 * t$$
$$x_L = 0.08539303 + 0.004226435 * t$$
$$y_L = 0.7478 - 0.000292166 * t$$

[4] `http://didaktik.physik.uni-essen.de/~backhaus/Venusproject/stuff/Tabelle.xls` The output format of `Bildauswertung.exe` makes the import to the worksheet easy.

(b) By using these linear fits, calculate better positions of Mercury for Udaipur and Teide at 9.15 UT (t=75 min) and derive the corresponding parallactic shift β!

$$\left.\begin{array}{l} x_U=0.40781 \\ y_U=0.72310 \\ x_T=0.39743 \\ y_T=0.71743 \end{array}\right\} \quad \Longrightarrow \quad \beta_{UT} = 11.2''$$

(c) As in exercise 3, derive now a (hopefully!) better measure of the solar parallax!

$$\pi_S = 11.4''$$

(d) Repeat these steps for the comparison between Learmonth and Teide at 9.15 UT! You will find a better measure of the solar parallax probably because of the larger distance between the sites.

$$\left.\begin{array}{l} x_L=0.40238 \\ y_L=0.72589 \end{array}\right\} \quad \Longrightarrow \quad \beta_{LT} = 9.3'' \overset{\Delta=1.85,\ w=62.6^\circ}{\Longrightarrow} \pi_S = 7.1''$$

Exercise 5 To get Mercury's Learmonth position at 9.15 UT the data must be extrapolated by 1h45min. There is no common observation time for this two observatories but it seems to be better to choose a moment as close to the both intervals of observation as possible: 7.45 UT!

Which measure for the solar parallax do you find by comparing the Learmonth and Teide positions of Mercury at this moment?

$$\beta = 12.7'',\ \Delta \sin w = 1.83 \quad \Longrightarrow \quad \pi_S = 8.6''$$