The Diurnal Parallax of the Moon
Example: November 28/29th, 2012

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The advantage of determining the distance to the Moon by measuring its diurnal parallax is that one doesn't need a distant partner for simultaneous observations. In this paper this method is demonstrated with position data calculated with computer programs. The goal of this article is to animate the reader to execute the parallax observation in reality.

1 Measurements

The geographical position of the observer is assumed to be $(\varphi, \lambda) = (50°, 10°)$.

In the night November 28/29th the topocentric equatorial coordinates of the Moon are measured twice, for instance by photographing it and measuring its angular distances to bright stars in the neighbourhood.

1. 19:00 UT: $\alpha_1 = 4h30m48s = 67.70°,\ \delta_1 = 20°01'48'' = 20.03°$
2. 5:00 UT: $\alpha_2 = 4h47m11s = 71.795°,\ \delta_2 = 20°20'16'' = 20.34°$

In order to measure the proper motion of the Moon a third position measurement is done one „moon day" (24h50m) later:

3. 19:50 UT: $\alpha_3 = 5h23m13s = 80.80°,\ \delta_3 = 20°20'16'' = 20.34°$

2 Determination of the parallactic effect

At the times of measurements 1 and 3 the relative positions of the Earth's center, the Moon and the observer are the same. Therefore, (nearly) no parallactic effect must be taken into account when comparing both Moon's positions and the averaged speed of the Moon's proper motion can easily be calculated:

$$\Delta\alpha = \alpha_3 - \alpha_1 = 13.10° \quad \Longrightarrow \quad \dot{\alpha} = 0.528°/h \tag{1}$$

$$\Delta\delta = \delta_3 - \delta_1 = 0.31° \quad \Longrightarrow \quad \dot{\delta} = 0.012°/h \tag{2}$$

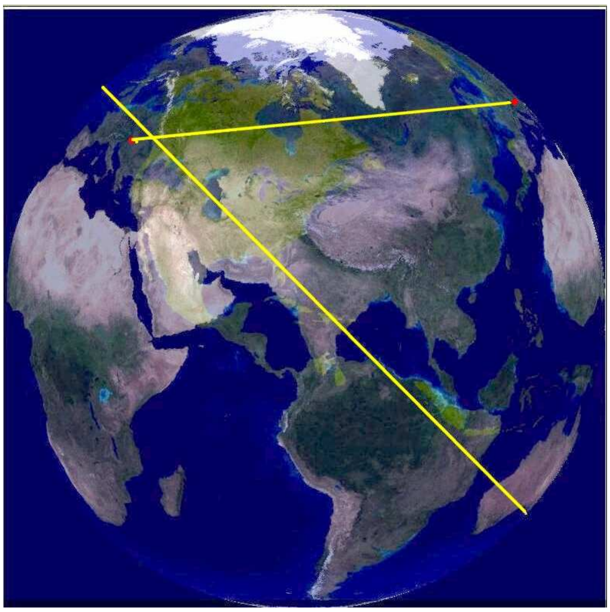

Figure 1: The two positions of the observer relative to the Moon and their (projected) distance with respect to the diameter of the Earth

Assuming the proper motion to be of constant speed the ,,parallax free" position (α_2', δ_2') of the Moon at time t_2 can be calculated:

$$\alpha_2' = \alpha_1 + \dot{\alpha}(t_2 - t_1) = 72.98^\circ \quad (3)$$

$$\delta_2' = \delta_2 + \dot{\delta}(t_2 - t_1) = 20.15^\circ \quad (4)$$

The parallactic effect between t_1 and t_2, therefore, is:

$$\Delta\alpha_p = \alpha_2 - \alpha_2' = -1.18^\circ \quad (5)$$

$$\Delta\delta_p = \delta_2 - \delta_2' = 0.07^\circ \quad (6)$$

$$\Longrightarrow \quad \Delta p = \sqrt{(\Delta\alpha_p \cos\delta)^2 + (\Delta\delta_p)^2} = 1.11^\circ \quad (7)$$

This angle of parallax is more than twice as large as the Moon's angular diameter!

3 Determination of the distance to the Moon

When determining the Moon's parallax by simultaneously measuring its topocentric coordinates the base line of the measured parallax is the distance between both observers as seen from the Moon. When the same observer measures the Moon's position twice at the same place but at different times he is ,,transported" to the second relative position to the Moon by the Earth's rotation. Because the Moon has moved in the meantime the second measured position must be corrected by the parallax effect determined above.

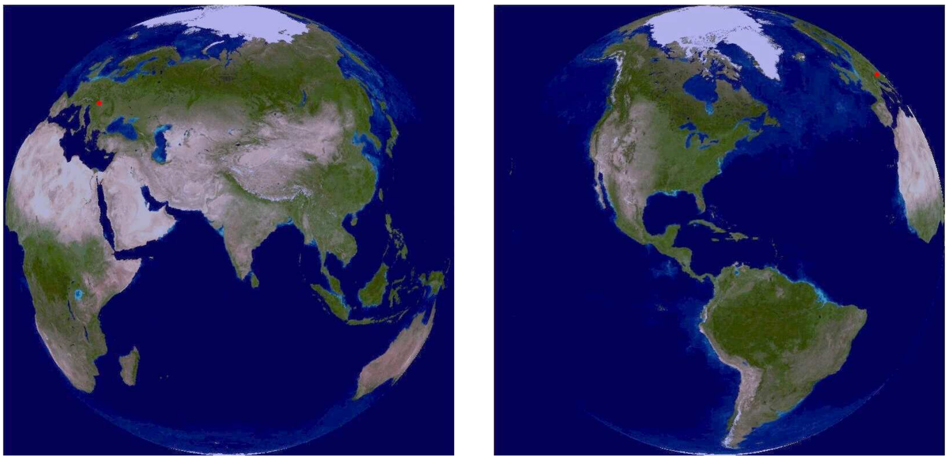

Figure 2: The night side of the Earth and the positions of the observation site as seen from the Moon at 19:00 and 5:00 UT

3.1 Rough but vivid method

With the program **HomePlanet**[1] it is possible to look to the Earth from the Moon (fig. 2). In this way it is possible to ,,observe" the relative position change between the times of observation[2]. By superposing both pictures (fig. 1) it is possible to measure the baseline length l of the parallax measurement:

$$\left.\begin{aligned} d(Obs_1, Obs_2) &= 500px \\ d(Earth) &= 781px \end{aligned}\right\} \quad \Longrightarrow \quad l = 1.28 R_E \tag{8}$$

With the parallactic effect Δp derived above (eq. 7) we finally get the distance d_M to the Moon:

$$d_M = \frac{l}{\tan \Delta p} = 62.3 R_E \tag{9}$$

The true geocentric distance in this night is $dM = 63.7 R_E$.

3.2 Algebraic calculation

The usual way of determining the geometrical parallax of an celestial object is to measure its topocentric position simultaneously from distant sites. This has been done in several projects: ,,Simultaneously Photographing of the Moon and Determining its Distance"[3],

[1] http://www.fourmilab.ch

[2] To be exakt the second picture must be ,,observed" from the first position of the Moon (fig. 2). Even that is possible with **HomePlanet** but the little difference doesn't matter for this method.

[3] `http://www.didaktik.physik.uni-due.de/ backhaus/moonproject.htm`

,,IYA2009: The distance to the Moon"[4] and ,,Measuring the Distance to the Sun"[5]. The correct algorithm for the calculation has been published therein[6].

It is possible to use that algorithm for the evaluation of observations described here in the following way: If one adds the parallactic displacement in equations (5) and (6) to the observed topocentric position (α_1, δ_1) one will get the position the observer would have measured at time t_2 if the proper motion of the Moon had been zero.

$$\alpha_2'' = \alpha_1 + \Delta\alpha_p = 65.96^\circ \tag{10}$$
$$\delta_2'' = \delta_1 + \Delta\delta_p = 20.10^\circ. \tag{11}$$

With this interpretation we get two ,,simultaneously" measured positions of the ,,fixed Moon" (α_1, δ_1) and (α_2'', δ_2'') which can be evaluated by using the former algorithm. For instance, this may be done with the worksheet `diurnalParallaxe.xls`. The result is

$$61.9 R_E \leq d_M \leq 62.3 R_E \tag{12}$$

3.3 Combination of many measurements during the night

When the parallactic effects are calculated for several position measurements during the same night the graphical representation of the results will show an ellipse like figure as can be calculated and drawn with the program `Mondparallaxe`[7] (fig. 3).

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[4] `http://www.didaktik.physik.uni-due.de/IYA2009/IYA2009-MoonsParallax.html`

[5] `http://www.eso.org/public/outreach/eduoff/aol/market/collaboration/solpar/`

[6] `http://www.didaktik.physik.uni-due.de/IYA2009/IYAParallaxe.pdf`, in German only, and `http://www.eso.org/public/outreach/eduoff/aol/market/collaboration/solpar/solpar-det.html`, in English

[7] `http://www.didaktik.physik.uni-due.de/ backhaus/AstroMaterialien/intern/Programme/ Mondparallaxe.zip`

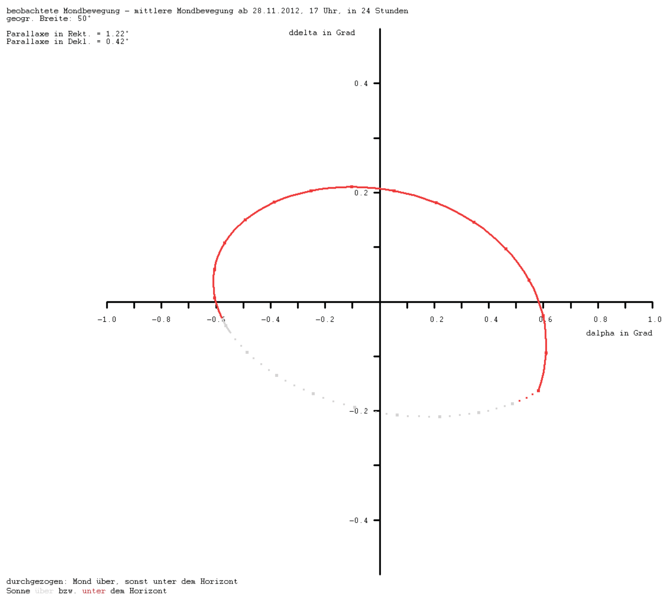

Figure 3: The parallactic shifts of the Moon measured during one night form a figure which can be interpreted as parallactic ellipse formed by the motion of the observer due to the Earth's rotation